Teaching the EPR–Paradox at High School ?

Gesche Pospiech
Institut for Didaktik der Physik
Johann Wolfgang Goethe University Frankfurt

February 2, 2008

Abstract

The discovery of quantum mechanics in the beginning of our century led to a revolution of physical world view. Modern experiments on the border of the classical and the quantum regime made possible by new techniques open better insight and understanding of the quantum world and have impact on new technological development. Therefore it seems important that students and even pupils at higher grades become acquainted with the principles of quantum mechanics. A suitable way seems to be given by treatment of the EPR–gedankenexperiment.

0.1 Introduction

The first question to be answered is: why should quantum theory be taught at school at all? For choosing this topic there are the following three reasons:

1. Quantum theory is the fundamental theory of modern physics. It plays a significant role in nearly all modern developments of physics. Many recent experiments and research in nanostructures with large applicability in technology rely on quantum effects.

2. Quantum theory has important philosophical aspects. Many people are highly interested in interpretation and understanding quantum theory as shows up in the many popular books about this subject.

3. Pupils at the age from 16+ on are searching for their place in the world. They are trying to understand the world and are open for philosophical hints that help them in building their own world view.

But often this highly fascinating subject is avoided at school because of mathematical and conceptual difficulties. I therefore want to show a possible way to introduce quantum theory in a manner suitable for interested pupils. In this article I concentrate on the mathematical part because here lie some difficulties. For an introduction into the philosophical aspects I have developed a dialogue between philosophers from different times - classical antiquity (Parmenides), the Enlightenment (Kant) and from our century, published elsewhere, ([Pos98]). The goal of the following is to clarify the main difficulties in teaching the physical basis of quantum theory and how to keep them minor.
Speaking qualitatively about quantum theory in an adequate way is nearly impossible in itself since all our concepts and terms have been developed along everyday experience. Hence our language is well suited to communicate about concrete physical objects with well determined properties or about psychological issues. In my opinion this last property should be used in dealing with the interfering and superposing objects of quantum theory that may have more similarities or associations with pyschological feelings than with concrete balls or waves occurring in classical physics. In every attempt to talk about quantum theory one has to be aware of this principal difficulty already recognized by Bohr, Heisenberg, Pauli and others. In the complementary worlds of the quantum regime and the classical regime we only are at home in the classical regime. The other regime remains accessible only through sophisticated experiments - even if an experimentalist would call them easy and simple.... One way out might be to talk in images - but soon one arrives at poor analogies. Hence, in order to reach more than only a superficial knowledge at least some hints to the mathematical background of quantum theory must be given. On the other hand at least in the beginning some themes should be avoided to facilitate the pupils the understanding of the peculiarities of quantum theory.

0.2 What can be done without too many technicalities?

It is nearly impossible to understand quantum theory without considering its mathematical structure. Nevertheless at school the mathematical apparatus of quantum mechanics has to be abandoned for the major part. The main ideas, however, can be presented quite easily with help of the typical quantum phenomenon "spin" having no classical analogues. Experiments with polarized photons may help in conveying the essentials. In the following I describe the reasons for taking spin as the first subject in treating quantum theory in more detail. Furthermore I explain with the example of EPR–gedanken experiment how to proceed.
Treating the phenomenon "spin" right in the beginning has several advantages:

- Spin lies at the heart of quantum theory. Its properties are used to explain the different statistics, the fine structure of spectra, the splitting of spectra in external (electrical or magnetic) fields. Already a spin system consisting of two particles, i.e. living in a four–dimensional Hilbert space can no longer be described classically. This proof is similar to the proof of Bell inequality, ([Bau]).

- The procedure to describe spin mainly by its structure is typical of quantum theory. Furthermore, the mathematics of spin is quite simple, using mainly the well–known Pauli– matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

  With the help of these quite simple looking matrices, acting on two–dimensional Hilbert space, the most essential mathematical structures of quantum theory can be explained and interpreted, see table 1. Some details are explained in the next section.

- The meaning of Heisenberg uncertainty relation can be explained as principal non–existence of fixed values for properties hence of their non–determination, [Pos99]. Therefore the danger that the uncertainty is perceived as measurement mistakes can be drastically diminished. Some helpful constructions even can be visualized on the blackboard.

- Spin is a phenomenon of special importance in modern experiments reaching from Nuclear Magnetic Resonance used in medical applications to realizations of the Einstein–Podolsky–Rosen gedankenexperiment. Its treatment opens the way to a discussion of philosophical aspects of quantum theory which quickly reaches the main points: the question of reality and objectivity in nature treated on a mathematical and physical foundation.

0.2.1 An Example

As an example I show how the arguments of Einstein, Podolsky and Rosen in their famous paper [EPR35] can be used to show the power of the mathematical formalism and - even more important - how the mathematical constructions can be interpreted in this framework. This allows a bridge to be built from the mathematical structures over the physical phenomena and connecting to a philosophical discussion. Instead of arguing very sophisticated within the mathematical formalism the main goal should be to uncover the main aspects of quantum theory and in this way to build a solid fundament from which the mathematics can be developed further, (see also table 1).
The argumens of EPR can be developed the following way:

Step 1: The mathematical tools In 1935, the year of the EPR-paper the mathematical framework has just been settled implying the following main points:

- The *state* of a quantum object is given by a state vector ψ containing all the available information, i.e. a complete description of the physical properties of the quantum object.
- Each *physical quantity* is given together with all the possible results of a measurement of that quantity and corresponding eigenstates, i.e. all the states a quantum object can attain after a measurement. A mathematical realization of this concept is given for instance by matrices.
- Arbitrary states can be expressed with aid of the eigenstates of such a matrix resp. physical quantity.

In a deviation from the original argument of EPR I would advise taking the spin realized with the above–mentioned Pauli–matrices as a concrete example. The students can compute the eigenvalus and eigenstates easily from the matrices. The possible measurement results (eigenvalues) are $+1$ and -1 together with the corresponding eigenstates. The first expriment showing this property directly has been the Stern–Gerlach–experiment. Furthermore the Pauli–matrices fulfill the condition crucial for the next step of argument of EPR: they do not have any eigenstates in common. Hence there always are several possibilities to represent the spin state of a quantum object, namely with respect to the respective eigenstates of the different matrices correponding to the spin di rections. The representation of an arbitrary spin state $\psi(s)$ with respect to the eigenstates of σ_x would be

$$\psi(s) = c_1 \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + c_2 \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Mathematical term	**Physical Interpretation**	**Example**
vector	physical state	$\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
operator	physical quantity	$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
eigenvalues of an operator	possible results of measurements	$+1, -1$
eigenstates of an operator (normalized to 1)	physical states with a fixed value for the physical quantity in question	$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$
vector addition	superposition (no fixed value for the physical quantity in question)	
development into eigenstates	representation of arbitrary physical states with respect to the corresponding physical quantity	$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \right]$
coefficients (squared) of development	probability of getting the corresponding measurement result	probability of getting either $+1$ or -1: $\left(\frac{1}{\sqrt{2}}\right) **2 = \frac{1}{2}$

and with respect to the eigenstates of σ_z the same state ψ would look like:

$$\psi(s) = k_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + k_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with different coefficients $(c_i) \neq (k_i)$. (For a concrete example look at the table.) This fact perhaps does not matter too much since we always can change coordinates (here it would be the rotation of a coordinate system by an angle of 45 degree). But here it means that the spin state of a given system possesses two different representations belonging to the "same piece of reality"(EPR). The most interesting thing happens in the next step!

Step 2: The experimental setup Two quantum objects, e.g. photons, are brought into interaction or produced in a single process and hence become entangled i.e. they share a common "history". After that they are separated from each other without any further manipulation, let us say one is brought to the moon, the second stays on earth.
Because of their common "history" they are described by one common state ψ which is not just the addition of the states of the single photons. This consideration is central for the whole argument of EPR. The development of the entangled state of both photons into eigenstates with respect to eigenstates of σ_x is given by:

$$\psi(s_1, s_2) = \psi_1(s_1) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \psi_2(s_1) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

and with respect to the eigenstates of σ_z:

$$\psi(s_1, s_2) = \phi_1(s_1) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \phi_2(s_1) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The only difference to the representations above is that the coefficients now depend on s_1 The meaning of these two representations is that *photon 1 is described differently depending on the description chosen for photon 2*, namely $\psi_i(s_1)$ resp. $\phi_i(s_1)$. This is called the entanglement of the two photons. Therefore I would prefer to call the whole system consisting out of these two photons rather a "diphoton" in order to emphasize that they build *one whole* (also see step 5 below).

Step 3: Classical Assumptions Assuming a fixed objective reality and demanding that physics has to give a complete description of reality Einstein arrives at a contradiction to the predictions of quantum theory. More precisely, Einstein assumes:

1. Separability
   Classical Physics only knows action between objects in direct contact with

each other. With “object” in this sense I also denote e.g. fields. Hence if two objects are separated in space, including intermediating fields, all future manipulations on them are absolutely independent from each other. We could summarize this in the sentence: Spatially separated objects also are physically separated. This is an implicit assumption of EPR that is not spoken out directly, but is underlying the whole argument as can be seen in the last paragraph of the famous EPR-paper [EPR35].
“Separability” hence means that the respective descriptions of two spatially separated photons should be totally independent from each other.

2. Physical Reality
Einstein defines a pragmatic criterion for reality: Every well determined physical quantity has to have a representation in the theory. The point herein lies in the question: Which properties are well determined? Einstein regarded every physical quantity that can be measured as well determined. But quantum theory deviates in so far from classical physics as not all (in principle) measurable quantites have well determined properties at the same instant. They only possess them as a *potentiality*.

From this view point the different descriptions from above (step 2) should not occur in a “good” physical theory.

Step 4: Quantum Theoretical Outcome We can get information about the photons only after a measurement. What can possibly happen then? There are several possibilities (as an example):

1. The spin of photon 2 is measured in x-direction. At the same instant the spin state of photon 1 is $\psi_1(s_1)$ or $\psi_2(s_1)$ according to the result of the measurement at photon 2.

2. The spin of photon 2 is measured in z-direction. At the same instant the spin state of photon 1 is $\phi_1(s_1)$ or $\phi_2(s_1)$ according to the result of the measurement performed on photon 2.

That means that photon 1 immediately “knows” the **kind** of measurement done on photon 2 far away as well as its **result**. Einstein calls this a “spooky action at a distance”, which may not occur in classical physics.

Step 5: Interpretation The behaviour of both entangled photons is strongly connected to each other, they behave in spite of their spatial separation as one single quantum object. Therefore I propose to call these both a “diphoton” which suggests more clearly that there is only *one common state* of the whole system, and not an addition of states of separated photons. Furthermore, the outcome of measurements demonstrates that we may not assume that photon 1 or photon 2 had fixed values for their spin directions before measurement. For this purpose one could use the comfortable Dirac–notation for spin

states e.g.: $\psi(s_1, s_2) = |1,0\rangle - |0,1\rangle$ for an entangled spin state instead of the above used vector–notation. The Dirac–notation has the advantage of showing only the **relative** directions of spins of both photons, which is the only property that is fixed and well determined (in absence of manipulations). The directions themselves are not determined, they only show up **after** a measurement. Fixed values of properties do not exist in general, they only emerge in measurements. Once this essential point is grasped the way is open for applications.

This access consequently avoids possible pitfalls which in general erschweren understanding quantum theory.

0.3 Which Themes to Avoid in a First Approach?

From historical reasons, having their roots in the development of quantum theory, most ways of teaching the concepts of quantum physics refer to classical models. This "procedere" causes principal difficulties in understanding. Therefore every reference to classical concepts should be avoided as far as possible. The most important points to avoid are:

- **Speaking about position and momentum, i.e. about trajectories**
  If the concepts of position and momentum — well–known from everyday experience — are used at the very beginning of a course in quantum theory there exists the danger of transferring classical thinking to quantum theory, although everybody would say: clearly, in quantum theory there are no trajectories. Conceptual difficulties arising from use of the terms " position" and "velocity" can be avoided in the simplest manner if these fundamental classical concepts do not play any role in the beginning of a course in quantum theory. Then any association of classical ideas might disappear and students might recognize the philosophical significance of quantum theory far more easily. The most prominent example is the famous Heisenberg uncertainty relation for position and momentum which easily is misunderstood in a sense that the uncertainty simply relies on disturbance by measurement in the usual sense. Instead the uncertainty relations are kind of measure for distinguishing classical behaviour from quantum behaviour in that they determine whether two physical quantities can attain fixed values at the same instant. If two physical quantities can attain fixed values at the same instant the quantum object in question behaves "classically", if not it displays quantum behaviour as e.g. spin. Hence the role and the implications of the non–existence of fixed values for some properties at the same instant - as expressed in uncertainty relations - might not be fully appreciated in their revolutionary potential if one concentrates on "position" and "momentum". Besides undesired analogies to Newtonian mechanics the corresponding operators for position and momentum and their eigenstates are mathematically far more

difficult to handle than the 2×2–spin–matrices.

- **Speaking about particle–wave–dualism**
  Waves and particles both are classical concepts, complementary to each other. One could illustrate their relation by looking at the same object from different sights. E.g. a cylinder standing upright appears completely different from the above (a circle) compared to a look from the side (a rectangle). But this observation does not meet the essential point in quantum theory.
  A first step to avoid analogy to classical phenomena would be to use the term "quantum object" instead of wave or particle. Only after the quantum mechanical concepts are fixed there might be a careful use of those "classical" terms be allowed where unevitable. Perhaps the importance of using suitable terms may become clear with the example of the double–slit–experiment. If it is replaced by the so called Taylor–experiment in which photons display at the same time wave properties - they show interference - as well as particle properties - they arrive at distinct points on the film, the necessity of changing concepts gets far more obvious.

- **Speaking about spin as sort of spinning around**
  One should not give an image of spin. Especially one may not think in terms of an electron spinning around. The quantum mechanical spin is simply structure manifesting itself and its behaviour through experiments, especially in the Stern–Gerlach–experiment which may serve as an introductory experiment. As shown above the abstract structure of spin can de introduced to a certain extent, depending on the mathematical capabilities of students.

Those three points are mentioned here because their avoidance breaks with the tradition of teaching and speaking about quantum theory. The preceding sections showed an alternative.

0.4 Conclusion and Perspectives

The recent EPR–experiments are the starting point for all the current developments concerning the fundamentals of quantum theory as well as technological utopies in the area of quantum computing and teleportation. In addition it widely opens the door to philosophical discussions. In so far the EPR–gedanken experiment lies at the heart of quantum theory and its interpretation.
The entrance to quantum mechanics with help of the phenomenon spin quickly gives gifted or interested pupils a possibility of discussing the properties of quantum objects, the mathematical structures and the interpretation of quantum mechanics on a technically very modest level but nonetheless quite precise. As the spin inevitably is a purely quantum mechanical phenomenon this opens

via the EPR–gedanken–experiment a short way into the crucial points of understanding concepts as well as philosophical implications of quantum theory and hence gives the possibility for people to revisit their view of nature, their *Weltbild*. I regard this an important contribution to general education.

References

[Bau] Hellmut Baumg"artel. Mathematische Grundlagen der Quantentheorie. manuscript, http://www.math.uni-potsdam.de/mp1/vorlesun.htm.

[EPR35] Albert Einstein, Boris Podolsky, and Nathan Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 48:696–702, 1935.

[Pos98] Gesche Pospiech. Vom Atommodell zur Quantenphysik und zur"uck. In Fachverband Didaktik der Physik Deutsche Physikalische Gesellschaft, editor, *Didaktik der Physik – Beitrge zur 62. Physikertagung Regensburg 1998*, pages 178–183, 1998.

[Pos99] Gesche Pospiech. Spukhafte Fernwirkungen in der Quantentheorie? *Physik in der Schule*, 37:56–59, 1999.